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On the use of model of growth of knowledge by repetition in didactics experiments

Abstract

The article focuses on the describing of the models of growth of knowledge. We describe different models and we point out their common features and the differences. Based on experimental data we show using of these hypothetical models to describe real results. We analyse how and why the model curve fits collected data.

The result is an overview of existing models and their comparison with the model of growth of knowledge by repetition (MGKR) and its use in experiments. We focus on the dependence of growth of knowledge on the complexity of the observed structures in terms of the number and the interdependence of autonomous units intervening in problem solving.

Keywords: model, growth of knowledge, experiments, theory of teaching

Introduction

Currently it places great emphasis on the effectiveness of human activities in all scientific fields. Theory of teaching is not an exception. The educational process is strictly limited in time. It led many of researchers to seek ways how to get the best effect in minimal time. Models of increase of the knowledge would help to show the effectiveness of different teaching methods in the education process.

Models of growth of knowledge by time

Teaching is primarily based on the idea that growth curve follows the rules of biological base of creating synapsis connections called Hebbian rule, which can be mathematically expressed by equation:

$$y = \left(1 - e^{-\lambda t}\right),$$

where the parameter ***t*** represents the duration of time of the learning process, so

$$0 \leq t.$$

The parameter λ is generally accepted as the observed individual's ability to learn. The fact is that λ is the only variable that distinguishes different learning curves. However, it means that the significance of the parameter λ involves several factors (eg. the method of the learning, difficulty of the curriculum, external influences of the learning process), not only personal conditions of pupil.

Brain structures, according to research in biology and neuroscience, are not formed continuously over time. According to knowledge and research, arising structures are stabilized by repeating [Fields, 2005a]. Any repetition of knowledge structure strengthens its stability and permanence. And therefore following iteration of knowledge structure is built on a stable base of structures.

Another reason is that mentioned curve describes, in the first approximation, learning of simple knowledge or the creating of a single autonomous structure in the brain. The real process is more complicated. In the real course is not created only one structure, but several independent or even interconnected autonomous structures. As stated in [Lacsný, 2005]: "We define autonomous units as a minimal structure of the brain, which should be to handle mental operation."

Gamble's model corrects deficiency of above mentioned model [Gamble, 1986]. Gamble's model was one of the most recognized models that were constructed in the field of didactics and theory of learning. His model constructs generally accepted exponential growth curve of knowledge in a few simple assumptions. As it is described in [Blasiak, 2011] Gambe assumes that learning is proportional to teaching, and also that learning is increase (change) of knowledge at the time:

$$L = Z.T$$

and

$$L = \frac{dK}{dt},$$

where ***L*** is the expression of learning, ***K*** is for knowledge, ***T*** is for teaching, and ***Z*** is the expression of coefficient of the individual properties of pupil.

If we limit the maximum possible level of knowledge

$$K_{\max}$$

that can be achieved by learning, and if we deploy a constant coefficient, which will constitute learning intensity, labeled ***C*** we get

$$\frac{dK}{dt} = C.K.(K_{\max} - K).$$

The solution of this differential equation is an expression called a logistic curve or learning curve also:

$$K(t) = \frac{K_{\max}}{1 + (\beta - 1)e^{-K_{\max}.C.t}}$$

where

$$\beta = \frac{K_{\max}}{K_0},$$

where K_0 is the level of the knowledge at the beginning of the learning process (t_0=0). This basic level of knowledge K_0 cannot be zero.

Model of growth of knowledge by repeating

In previous models, it is necessary to consider the growth of knowledge in the time. However, experiments are constructed and based on repetition of problems solving. The solution of the time factor problem offers a model of growth of knowledge by repeating (MGKR) designed and constructed by Boris Lacsný from Constantine the Philosopher University in Nitra.

As stated in [Lacsný, 2005]: *In contrast to the traditional exponential growth model, our model treats time flexibly within certain limits (time delays), and so come into the spotlight the number of repetitions and not the time.* As stated by Fields, the time delays vary for long-term memory in the range of 6-45 minutes, depending on various factors. [Fields, 2005b]

The MGKR, which just like previous models describes an exponentially growing curve, indicates as the probability

$$P(n)$$

of stabilization of newly formed structures after ***n*** repetitions. Mathematically expressed in equations:

$$P(n) = 1 - e^{-\alpha n} \equiv 1 - q^{n} = P_q(n),$$

where

$$e^{-\alpha} = q.$$

The meaning of the factor α can be interpreted as "efficiency" with whom the student is able to learn the curriculum, or otherwise as "modesty" of subject matter [Lacsný, 2005] In biological point of view it is a parameter of construction of autonomous unit in brain. The parameter α is in fact another form of the parameter λ from equation

$$y = \left(1 - e^{-\lambda t}\right)$$

or the parameter C from equation

$$K(t) = \frac{K_{\max}}{1 + (\beta - 1)e^{-K_{\max}.C.t}}.$$

The neurological structures, but also the whole learned structures are characterized by autonomous entities that interact and interconnect.

As stated in [Benko, 2011]: The ability to successfully solve problems requiring an autonomous unit A and successfully solve problems requiring the formation of an autonomous unit B is not the same as the ability to successfully solve the problems requiring the unification of these autonomous units. In other words, a combination of knowledge, requiring the creation of one complex structure of the two autonomous entities in the brain, needs more repetition – it is more time consuming. The diversity of these learning curves illustrated in fig. 1.

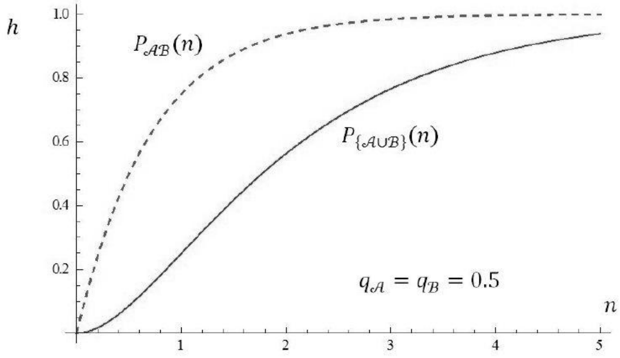

Fig. 1. Dependence of the growth curve of knowledge on the complexity of required structures [Benko, 2011]

The discussed model (MGKR) shows also the dependence of dynamics of the growth of knowledge on the number of autonomous units needed during the problem solving process. The probability

$$P(n; N_a)$$

of the completion of $\boldsymbol{N_\alpha}$ autonomous structures in $\boldsymbol{n}$ repetitions is then

$$P(n; N_a) = (1 - q^n)^{N_a},$$

where

$$(1 - q)$$

is the probability that one autonomous structure will be complete by one repetition [Lacsný, 2005]. The shape of the function

$$P(n; N_a)$$

for various $\boldsymbol{N_\alpha}$ is shown in fig. 2.

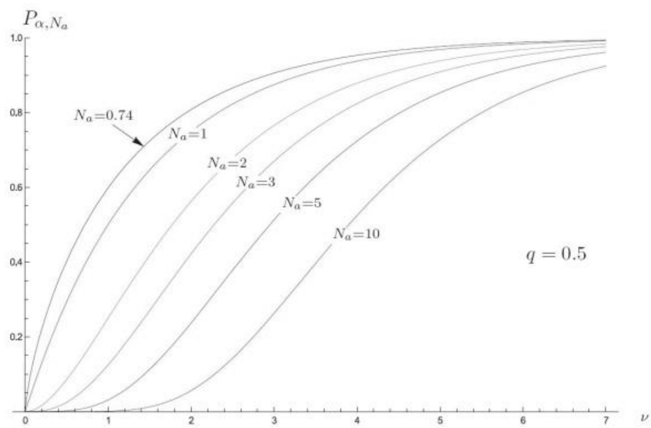

Fig. 2. Dependence of the curve of growth of knowledge on the number of autonomous units, needed during the problem solving process [Lacsný, 2005]

The MGKR model is therefore possible to describe multiple level complexities, where the first level is realized by autonomous structures.

Experiments and models of growth of knowledge

The exponential nature of the curves was confirmed in the research conducted on hundreds of respondents D. I. Nurminsky. Curve mathematically expressed by this model corresponds to the course and shape of the result of performed research (fig. 3). The ability to solve problems properly was observed depending on the number of repetitions of curriculum. It is application level of cognitive processes. The research shows that repetition of the curriculum increases the percentage of correct answers of pupils.

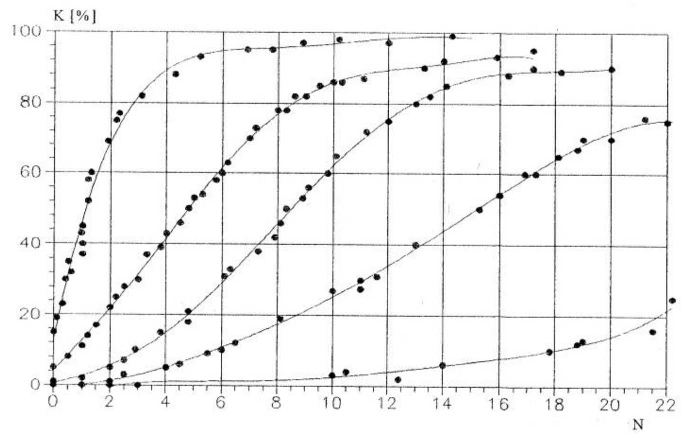

Fig. 3. Results of research of growth level of knowledge by repeating realized by Nurminsky. The results are curves corresponding to MGKR. Taken from [Błasiak, 1996]

We can fit knowledge grow curves fit by various equations depending on mentioned models. As we can see on fig. 4, Hebbian rule model (on the bottom right) doesn’t fit well. It is caused by fact that experiment is aimed on more complex structures, as is memorizing. In contrast Gamble´s model (on the bottom left) and MGKR (at the top right) fit with high correlations.

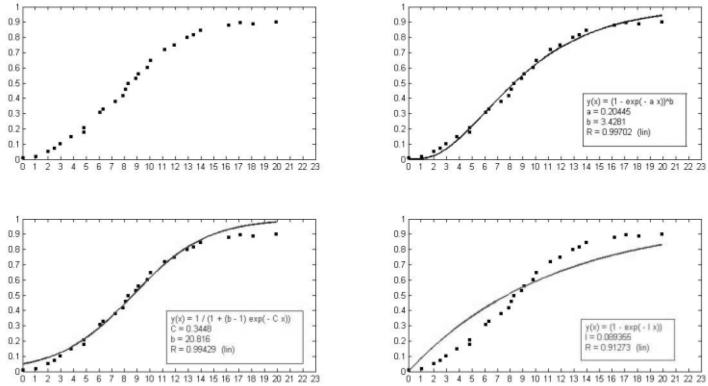

Fig. 4. Fitted experimental results. Top left are experimental data, top right is a fitting curve by MGKR, bottom left is the fitting curve according to the Gamble model and bottom right is the fitting curve according to Hebbian rule

Data were fitted by mathematical program MatLab, using of licenced toolbar EzyFit. As stated authors on its webpage [http://www.mathworks.com/matlabcentral/fileexchange/10176-ezyfit-2-40/content/ezyfit/html/ezyfit_faq.html#a11, 2012]: „*The core function of the EzyFit toolbox is ezfit, which is based on Matlab's built-in FMINSEARCH function (Nelder-Mead method) […] The additional function show fit simply calls ezfit with graphical output.*“

As is described in [Benko, 2012] the increase in the level of knowledge by repeating was seen in the Rosiek's and Błasiak's experiments in Pedagogical University of Cracow. They have observed an increase of success of remembering depended on repetition. Although the experiment deals with short term memory and less complex cognitive process – memorizing the result is well-describe by the curve of growth of knowledge. It can be described even by simplified shape of the Hebbian rule curve, because it has been creating only one autonomous structure in the brain.

Observed was one sign, twenty-digit number, which was showed to students for a short time (order of seconds) and after preview the students were asked to write down the number. There were no digits while students wrote down number. They could see only colored fields of digits. Green fields correspond to success and red ones were for incorrect answers. After entering all the number, the correct digits were shown again. Digits highlighted in red were entered incorrectly. After a short preview the student was re-invited to entry number again. The procedure was repeated few more times.

It is obvious that the pupil progressively memorized all the digits of number. The growth of this remembering was affected by various factors, including personal physiological and psychological assumptions of the respondent, by the disturbances of environment, by the time while the number was displayed, and others.

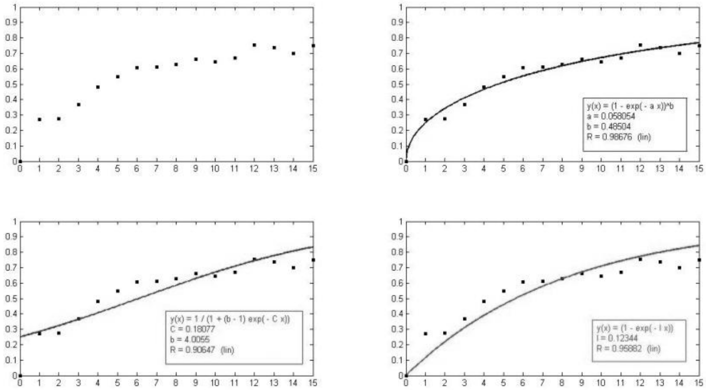

Fig. 5. Fitted results of WinMemo experiment. Top left are experimental data, top right is a fitting curve by MGKR, bottom left is the fitting curve according to the Gamble model and bottom right is the fitting curve according to Hebbian rule

Fitted grow curves by various equations depending on mentioned models (fig. 5) can show that Hebbian rule model (on the bottom right) fit well, just like MGKR. It is caused by fact that experiment is aimed on less complex structures. In contrast Gamble´s model (on the bottom left) doesn't fit with high correlation.

The conclusion

Descriptive models of growth of knowledge refer to the dependence of this growth over time. However, experiments pointed to dependence of growth of knowledge by repetitions. It is therefore important to consider the using of the model to follow experiments and explain them the right way.

It is also important to interpret correctly the complexity of the observed structures. Many models work well in the first approximation if we observe only one unit, a one complete autonomous structure. This is sufficient in the lowest (and least complex) cognitive processes, such as memorizing. It is not sufficient in the complicated processes of learning entered into by number of dependent (or independent) autonomous structures.

And so MGKR becomes unique model using the number of repetitions in its mathematical interpretation. The interval function values correspond to the needs of the theory of teaching. Because of many experiments we do not exclude the existence of a related model based on similar principles.

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Referencies

Benko M.: *Use of application-level growth model*. [In:] Young Researchers 2011, PhD students, Young Scientists and Pedagogues Conference Proceedings, volume 12, Nitra, 2011. Faculty of Natural Sciences, Constantine the Philosopher University in Nitra.

Benko M.: *On the status of the model of growth of knowledge by repetition on the field of didactics*. [In:] 13-th international scientific conference of PhD students, Young Scientists and Pedagogues, volume 13, Nitra, 2012. Faculty of Natural Sciences, Constantine the Philosopher University in Nitra [in print]

Błasiak W.: *Marzenia o teorii nauczania.* Impuls, Kraków 1996

Błasiak W.: *Rozważania o nauczaniu przyrody*. Wydawnictwo Naukowe UP, Kraków 2011

Dewey R. A.: *The Learning Curve*. http://www.intropsych.com/ch07_cognition/learning_curve.html

Fields R. D.: *Making memories stick*. "Scientific American", 2(292):58–63, 2005a.

Fields R. D.: *Erasing memories*. "Scientific American", 4(16):29–35, 2005b.

Gamble R.: *Cognitive momentum*. Phys. Educ. 21, 1986

Hassan M. M.: *On quantitative mathematical evaluation of long term potentiation and depression phenomena using neural network modeling*. 2005. International Conference on Simulation and Modeling. Eds.: V. Kachitvichyanukul, U. Purintrapiban, P. Utayopas

Hassan M. M.: *On Simulation of e-Learning Convergence Time Using Artificial Neural Networks*. Banha University, Egypt 2009. Currently with Arab Open University (Kingdom of Saudi Arabia Branch)

http://www.mathworks.com/matlabcentral/fileexchange/10176-ezyfit-2-40/content/ezyfit/html/ezyfit_faq.html#a11 []dostęp 2012

Lacsný B.: *Matematické modelovanie fyzikálnych veličín*. Písomná práca k ústnejskúške, Univerzita Konštantína Filozofa v Nitre, Fakulta prírodných vied, Nitra, máj 2005.